Astrographic Chart: 20 charts, presented by the Royal Observatory, Greenwich, and 19 charts presented by the San Fernando Observatory. Photographs of the spectrum of Mira Ceti, presented by the Rev. W. Sidgreaves.

On the Possibility of Improving the Places of the Reference Stars for the Astrographic Catalogue from the Photographic Measures. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. On each of the plates taken for the Astrographic Catalogue there are certain stars of which meridian observations have been made, and the constants of the plate are found by using these recorded places. But the places are often defective, from errors of observation and accumulated proper motions; and the errors are indicated by the residuals found on comparing the photographic measures (reduced with the plate-constants found by using all the stars) with the individual meridian places. But these residuals cannot be taken as satisfactory corrections to the adopted places, because the plate-constants, having been found from faulty meridian places, are themselves faulty.

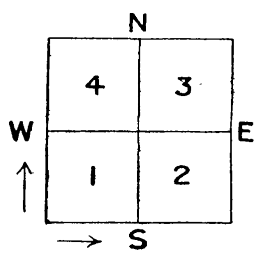

Fig. 1.

2. Taking only a single plate, if we correct the adopted places by the residuals found, and then solve for plate-constants again, we shall get precisely the same constants as before, and the residuals now will be all zero; but the improvement is of course only fictitious, and no real advance has been made. We need not, however, restrict ourselves to a single plate. Every star occurs on at least two plates, and we get at least two different residuals for it. Moreover, the stars in the four quarters of any plate will, as a rule, be on four different overlapping plates. Shall we then adopt as corrections to the original places the means of the residuals for each star (usually two, but sometimes more), and then determine the plate-constants afresh? There is no *prima facie* objection to this course; but it is questionable whether it is the best possible, for the following reason.

3. Call the residuals determined from the plate itself A, and those from other plates B: it is proposed to use $\frac{1}{2}(A+B)$. Now the portion $\frac{1}{2}A$ is simply non-effective. It has been already remarked

that if the residuals A be applied to the original places, and the constants redetermined, they will be the same as before; in other words, if we determine the *corrections* to a, b, c by solving a series of equations such as

$$\Delta a.x + \Delta b.y + \Delta c = \mathrm{A}$$

we shall get $\Delta a = 0$, $\Delta b = 0$, $\Delta c = 0$. Naturally we shall get the same zero result if we write $\frac{1}{2}$A instead of A on the right of all the equations. Hence it may be argued that we may as well save ourselves the trouble of introducing the system of residuals A into the equations at all, since they do neither good nor harm; with one possible qualification. Some stars occur on more than two plates, and if we form the mean of all plates, the residuals A will have a factor $\frac{1}{3}$ or $\frac{1}{4}$ in some cases instead of $\frac{1}{2}$. But there is clearly no systematic advantage to be gained by such exceptions, and we need not discuss them in detail.

4. The question therefore arises whether it is not better to exclude the residuals A altogether, since they do neither good nor harm in improving the plate-constants. Should not the corrections to the originally adopted places be determined from residuals B entirely?—*i.e.* instead of neglecting the $\frac{1}{2}$A as useless, and retaining the $\frac{1}{2}$B, should we not substitute B simply? or the mean of several B residuals if they are available? The answer to this question can only be arrived at by considering in some detail what the residuals represent.

5. For each star we write down two equations of the form

$$ax + by + c - \mathrm{X} = 0, \quad dx + ey + f - \mathrm{Y} = 0$$

and it will be sufficient to consider the first of these equations only, since the procedure is the same for both. In this equation x and y are the co-ordinates of the star, a, b, c the plate-constants to be determined, X the difference between the measured and calculated standard co-ordinates of the star. There will be as many equations as there are known stars on the plate, and they may be solved by least squares or any equivalent process. At Greenwich and Oxford the labour of least squares has been avoided by taking the mean of all the equations for the S and N halves of the plate and for the E and W halves; *i.e.* the four equations

$$\begin{aligned}
\{[1] + [2]\}/(n_1 + n_2) &= 0 \\
\{[4] + [3]\}/(n_4 + n_3) &= 0 \\
\{[2] + [3]\}/(n_2 + n_3) &= 0 \\
\{[1] + [4]\}/(n_1 + n_4) &= 0
\end{aligned}$$

have been formed; where [1] represents the sum of all the expressions

$$ax + by + c - \mathrm{X}$$

in the SW corner of the plate marked 1 in fig. 1: and n_1 is the number of stars in this portion. Next, the constant c is eliminated

by subtracting the second equation from the first and the fourth from the third, and we get the pair

$$\{[1]+[2]\}/(n_1+n_2)-\{[4]+[3]\}/(n_4+n_3)=0$$
$$\{[2]+[3]\}/(n_2+n_3)-\{[1]+[4]\}/(n_1+n_4)=0$$

from which to determine a and b.

6. If the stars are equally distributed in the four quadrants, so that $n_1=n_2=n_3=n_4$, these equations reduce to

$$[1]=[3] \quad \text{and} \quad [2]=[4]$$

The discussion thus divides itself into two parts: first, the consideration of what happens when the stars are uniformly distributed; and secondly, the effect of irregular distribution.

7. We shall take uniform distribution first; and we may take the very simplest case of it, viz. when there are just four stars placed at the centres of the four quadrants. If the side of the plate be $4s$, the co-ordinates of these stars referred to the centre of the plate as origin will be (see fig. 1)

1	$x=-s$	$y=-s$
2	$x=+s$	$y=-s$
3	$x=+s$	$y=+s$
4	$x=-s$	$y=+s$

and the equations for finding a and b become

$$-2s.a-2s.b=X_1-X_3$$
$$+2s.a-2s.b=X_2-X_4$$

Thus

$$4s.a=-X_1+X_2+X_3-X_4 \quad . \quad . \quad . \quad (1)$$
$$4s.b=-X_1-X_2+X_3+X_4 \quad . \quad . \quad . \quad (2)$$

and for c we add all four equations, so that

$$4c=X_1+X_2+X_3+X_4\,. \quad . \quad . \quad . \quad (3)$$

8. The residual X_1 is reduced, with these values of a, b, and c, to

$$X_1-(ax+by+c)$$

which on putting $x=-s$, $y=-s$, and the values of a, b, c given above becomes

$$\tfrac{1}{4}(X_1-X_2+X_3-X_4)=+\mathrm{I}, \text{ say} \quad . \quad . \quad . \quad (4)$$

9. The residual for the third quadrant is also reduced to $+\mathrm{I}$

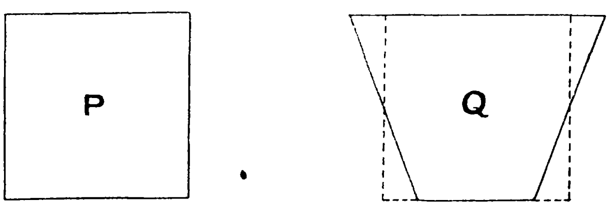

Fig. 2.

while the residuals for the second and fourth quadrants become $-\mathrm{I}$. This quantity I, therefore, cannot be removed by a linear solution.

Its effect is to alter the square P (fig. 2) into the trapezium Q; and it may be produced, not by erroneous adopted places, but by a "tilt" of the plate; or, in other words, by our assuming the wrong plate-centre to which to refer our standard co-ordinates. It is known that this modifies ξ, η into expressions of the form

$$\xi' = \frac{(1+a)\xi + b\eta + c}{1 - k\xi - l\eta}$$

$$= (1+a')\xi + b'\eta + c' + k\xi^2 + l\xi\eta \text{ approx.}$$

$$\eta' = d'\xi + (1+e')\eta + f' + k\xi\eta + l\eta^2 \quad \text{,,}$$

where (k, l) are the co-ordinates of the true centre. The terms which give rise to an I effect are the $\xi\eta$ terms, which are positive in the first and third quadrants and negative in the second and fourth. But it is known from experience of plates, in parts of the sky where stars are numerous, that no very large part of I is due to tilt in this way.

10. It is perhaps also as well to verify that the same I term would be given by the method of least squares, at any rate in the simple case at present under consideration. Solving the equations

$$\begin{aligned} -as - bs + c &= \mathrm{X}_1 \\ +as - bs + c &= \mathrm{X}_2 \\ +as + bs + c &= \mathrm{X}_3 \\ -as + bs + c &= \mathrm{X}_4 \end{aligned}$$

by least squares, the normal equations are

$$\begin{aligned} 4s^2a &= s(-\mathrm{X}_1 + \mathrm{X}_2 + \mathrm{X}_3 - \mathrm{X}_4) \\ 4s^2b &= s(-\mathrm{X}_1 - \mathrm{X}_2 + \mathrm{X}_3 + \mathrm{X}_4) \\ 4c &= (\mathrm{X}_1 + \mathrm{X}_2 + \mathrm{X}_3 + \mathrm{X}_4) \end{aligned}$$

which are precisely the same as those obtained in § 7. Hence the constants and residuals obtained are the same, and we still get the residuals $+\mathrm{I}$, $-\mathrm{I}$, $+\mathrm{I}$, $-\mathrm{I}$ for the four quadrants where

$$4\mathrm{I} = \mathrm{X}_1 - \mathrm{X}_2 + \mathrm{X}_3 - \mathrm{X}_4$$

11. It is convenient to have a name for the quantity I, and the name "inconsistency" will be adopted in the present paper for convenience. When I is not zero, *no* linear solution will fit the plate. We can find the value of I before applying any linear solution at all, since it is not altered by the application of linear terms.

12. Now it is clear that the procedure indicated in §§ 2 and 3, of taking the means between residuals from different plates, will not in general reduce the "inconsistency" for any plate to zero. It may reduce it, but even this is not certain. Calling the inconsistency of any plate I_0, and of the four overlapping plates

$+I_1$, $-I_2$, $+I_3$, $-I_4$, then the process of taking the mean of residuals substitutes for I_0 the quantity

$$\tfrac{1}{4}\{\tfrac{1}{2}(I_0+I_1)+\tfrac{1}{2}(I_0+I_2)+\tfrac{1}{2}(I_0+I_3)+\tfrac{1}{2}(I_0+I_4)\}$$
$$=\tfrac{1}{2}I_0+\tfrac{1}{8}(I_1+I_2+I_3+I_4)$$

The first term represents a certain gain, since I_0 is halved; but the second term may either increase or diminish the first numerically. On the average we shall perhaps not advance beyond halving the original inconsistencies. We can, of course, proceed to a second approximation and halve them again; and to a third. And by using the invariant property of the inconsistency we can make these approximations without performing the solutions on the plates if we simply have the inconsistencies for all the plates tabulated before us. It is even easy to summarise this process of approximation algebraically and combine several steps in one. But probably it will be better in practice to watch the process numerically. The end to be attained is the reduction of the inconsistencies to small quantities, zero for choice. And the question is suggested, Is there any simple process for making them all zero *en bloc*? If we knew the correct places of the stars and had perfect photographic measures, the inconsistencies would be all zero. The measures are not perfect, but the errors of measurement may be assumed small compared with the quantities given below; and the problem is to find a set of places for the standard stars to fit them as closely as possible.

13. We may still suppose the standard stars to be arranged in exact rows and columns, four on each plate, and each one common to two plates. Let us represent a set of advantageous corrections to the adopted places of these stars by the subjoined scheme:—

$$\begin{array}{cccccc} a_1 & a_2 & a_3 & a_4 & \cdots \\ b_1 & b_2 & b_3 & b_4 & \cdots \\ c_1 & c_2 & c_3 & c_4 & \cdots \\ d_1 & d_2 & d_3 & d_4 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{array}$$

so that the four stars on one particular plate, for instance, require the corrections b_2, b_3, c_2, c_3. Then if we reduce the inconsistency of every plate to zero we have a series of equations of the form

$$-b_2-c_3+b_3+c_2=4[B_2C_3] \quad . \quad . \quad . \quad (5)$$

where the symbol $[B_2C_3]$ is used to represent the inconsistency.

14. Let n be the number of plates, and therefore of such equations. There are four images on each plate, and therefore altogether 4 n star images; but since each star occurs on two plates, there will be only 2 n stars altogether. Hence we have n equations among 2 n quantities, and there are any number of ways of satisfying them. If we make a single assumption for each plate we get n new equations and all the corrections become determinate.

15. Suppose we make the assumption

$$b_2 + c_3 + b_3 + c_2 = 0 \quad . \quad . \quad . \quad . \quad (6)$$

that is, that the mean correction to the places on any plate is zero, or the plate-constant is undisturbed. This seems a natural assumption to try, at any rate in the first instance. Then, combining the two equations (5) and (6) for the same plate, we get

$$b_2 + c_3 = -2[B_2C_3] \quad (b_3 + c_2) = 2[B_2C_3] \quad . \quad . \quad (7)$$

that is, the inconsistency is divided equally between pairs of opposite quadrants.

16. Now if we follow the diagonal line of stars $a_1\ b_2\ c_3\ d_4$ we have a series of equations giving $a_1 + b_2$, $b_2 + c_3$, $c_3 + d_4$, and so on. Hence if we know any one correction a_1, we find successively $b_2\ c_3\ d_4$ and all the rest.

17. It is to be noted that the case is not the same for the adjacent diagonals in the same sense $b_1\ c_2\ d_3$ . . . . and $a_2\ b_3\ c_4$ . . . . There are, according to the scheme adopted for the Astrographic Catalogue, no plates running with corners in this direction—no plate $c_2\ c_3\ d_3\ d_2$ for instance. In covering the sky twice over, we select two only out of four possible ways of covering it; for considering any single plate $b_2\ c_2\ c_3\ b_3$, let us characterise it by the star at the left-hand top corner, in this case b_2. We can cover the sky completely by plates b_2, b_4, b_6, ... d_2, d_4, d_6 . . . . and so on, without using systems in which $b_3\ c_2\ c_3$ are represented at all. And by the scheme adopted for the Astrographic Catalogue, the overlapping plates would be the system c_3, including plates $c_1\ c_3\ c_5$ . . . . , e_1, e_3, e_5 . . . . but not including c_2 or b_3. The systems b_3 and c_2 would overlap the others in a different way. Had these additional plates been taken, we should have had $2\ n$ equations for inconsistency of type (5) instead of n only; and we could have determined the corrections to the star places completely from these alone, without any additional assumptions.

18. The absence of the plates of type b_1 and c_2 prevents the formation of equations for $b_1 + c_2$, $c_2 + d_3$, etc., but we get b_1 and c_2 from the diagonals of the other sense, $a_2\ b_1$ and $a_4\ b_3\ c_2\ d_1$, and so on. Hence if we know or assume all the a's, we can find all the other terms. Or similarly if we know or assume $a_1\ b_1\ c_1\ d_1$ . . . . we could follow the diagonals from these as starting-points and find all the quantities.

19. But it will throw light on the validity of the assumption of equations (6) if we take an actual example. We have hitherto been assuming an ideal distribution of stars, one and one only at the centre of each quadrant. In passing to an actual example, we must replace each ideal star by a number of actual stars scattered over the quadrant, and for a single residual we substitute the mean of several.

20. But these changes will be understood without further explanation, and the following example closely represents an actual

instance which occurred in the usual course of reduction of plates taken at Oxford in zone $+29°$, at R.A. $4^h\ 7^m$, where the stars are few in number. The original solution for plate-constants gave residuals for some stars differing by quantities such as $0{\cdot}006 = 1''{\cdot}8$ and $0{\cdot}009 = 2''{\cdot}7$ from the residuals found from adjacent plates. A new solution for plate-constants was accordingly made by least squares without removing these anomalies; and various assumptions as to error in scale value, etc. were tried without success.

21. In the light of the considerations above advanced, the inconsistencies of this plate and of overlapping plates were calculated with the following approximate results (the precise definition of

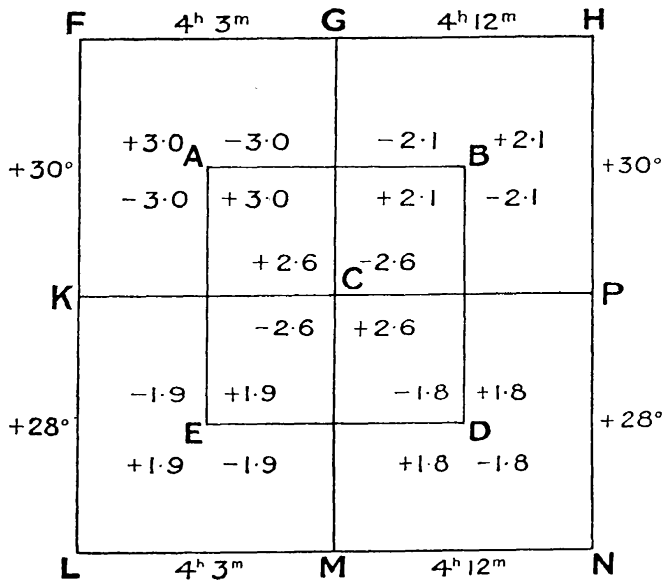

FIG. 3.

inconsistency for stars casually distributed has not yet been settled, and hence the results are provisional only):—

Zone $+30°$, For R.A. $4^h\ 3^m$ $I = -3{\cdot}0$, For R.A. $4^h\ 12^m$ $I = +2{\cdot}1$
Zone $+29°$, For R.A. $4^h\ 7^m$, $I = -2{\cdot}6$
Zone $+28°$, For R.A. $4^h\ 3^m$ $I = +1{\cdot}9$, For R.A. $4^h\ 12^m$ $I = +1{\cdot}8$

The unit for I is $0''{\cdot}3$ or $\cdot001$ of a réseau interval. It will be seen that the plates overlap by 4^{min} in one direction and 5^{min} in the other, and are thus not strictly divided into quarters; but let us neglect this for the present. The values for I indicate that the mean residuals (observed—adopted) in the quadrants are disposed as in fig. 3.

22. In the quadrant AC the agreement of the two plates is good, but in none of the other quadrants is it at all satisfactory. No linear solution can improve it, for such solutions do not alter I; and it will at once be understood why solving by least squares and altering the scale value by some assumption are equally unavailing.

23. We must begin by *assuming* some corrections to one set of outside quadrants. In default of other knowledge, and as an

illustration merely, let us assume *zero* corrections for the top row of quadrants FA, AG, GB, and BH.

Then the correction to AC must be $+6{\cdot}0$, in order to take up the $+3{\cdot}0$ and $+3{\cdot}0$ in the two quadrants FA, AC.

The correction to CD will be

$$+2{\cdot}6+2{\cdot}6-6{\cdot}0=-0{\cdot}8$$

and to DN $$-1{\cdot}8-1{\cdot}8+0{\cdot}8=-2{\cdot}8$$

Similarly following the diagonal HL we get corrections to

$$\begin{aligned} HB &= 0 \\ BC &= +2{\cdot}1+2{\cdot}1+0=+4{\cdot}2 \\ CE &= -2{\cdot}6-2{\cdot}6-4{\cdot}2=-9{\cdot}4 \\ EL &= +1{\cdot}9+1{\cdot}9+9{\cdot}4=+13{\cdot}2 \end{aligned}$$

The increase in this last set is alarming, and suggests a revision of the original assumptions; but we will first finish the example. The central plate is now to be revised as follows:—

We correct the adopted places of the stars in the four quadrants by the following quantities

$$\begin{array}{cc} +6{\cdot}0 & +4{\cdot}2 \\ -9{\cdot}4 & -0{\cdot}8 \end{array}$$

and then solve the equations for scale values and orientation again. The effects on the constants a and b are shown in equations (1) and (2) of § 7; that on c was arranged to be zero. But what immediately concerns us is that when this new linear solution is applied, the mean residuals in the four quadrants will now be zero; for we saw in § 7 that the effect of the corrections above tabulated, taken by themselves, is to leave us with four numerically equal residuals of the value

$$\pm\tfrac{1}{4}(-9{\cdot}4+4{\cdot}2-6{\cdot}0+0{\cdot}8)=\pm 2{\cdot}6$$

and these will just neutralise the residuals shown by the original solution. Indeed, it is the basis of the method to reduce these residuals to zero. The same will be true of other plates, and hence the discrepancies between adjacent plates will disappear.

24. This will only be true in the mean, *i.e.* at the centre of each quadrant. For stars away from these points errors in a and b will still introduce discrepancies, but they will be naturally smaller. One thing at least is clear: in default of some system of correction of the kind above indicated, there are bound to be discrepancies between adjacent plates in thinly starred regions, due to mistakes neither of measurement nor reduction, which cannot be removed by any change of the linear solution based on the original system of star places. To get improvement we must alter these places relatively to one another; and at first sight this seems alarming. But, after all, is it more drastic than any other modification suggested by the residuals?—taking the mean of the residuals for various plates, for instance?

25. As regards this latter process, we may note in passing how ineffective it may be in helping us to remove discrepancies between plates. Recurring to fig. 3 (p. 114) we see that the corrections suggested by taking the means from adjacent plates are as below, those suggested by observing the inconsistency being put alongside for comparison :

By "means"		By "inconsistency"	
+2·8	−0·3	+6·0	+4·2
−0·3	+0·4	−9·4	−0·8

The set obtained by "means" would leave us, after a new solution, with residuals of numerical value

$$\pm \tfrac{1}{4}(-0{\cdot}3 - 0{\cdot}3 - 2{\cdot}8 - 0{\cdot}4) = \pm 0{\cdot}9$$

for this plate, which is certainly less than the $\pm 2{\cdot}6$ we started with; but to get rid of the discrepancies altogether we apparently require quite a different set of corrections.

26. We return now to the assumptions made in deriving these corrections, which can obviously be improved. Let us first consider the effect of a different set of starting-points in the top row of quadrants. The diagonal H B C E L, for instance, gives us the series

$$0, \ +4{\cdot}2, \ -9{\cdot}4, \ +13{\cdot}2$$

which is becoming alarmingly large. How much of the increase is due to the assumption of zero as a starting-point? It is easily seen that if we start with $+a$, the effects on the series will be

$$+a, \ -a, \ +a, \ -a$$

and thus if we put $a = +6{\cdot}6$ we get the series

$$+6{\cdot}6, \ -2{\cdot}4, \ -2{\cdot}8, \ +6{\cdot}6$$

Thus, if the sum of one set of alternate terms in a diagonal differs from that of the other set, we can reduce the difference to zero.

27. Consider next the other set of assumptions represented by the equation (6) of § 15,

$$b_2 + c_3 + b_3 + c_2 = 0$$

If we make the alternative assumption

$$b_2 + c_3 + b_3 + c_2 = 4k \quad . \quad . \quad . \quad . \quad (8)$$

then combining with the equation for inconsistency

$$-b_2 - c_3 + b_3 + c_2 = 4[\mathrm{B_2C_3}]$$

we get

$$b_3 + c_2 = 2\{[\mathrm{B_2C_3}] + k\} \quad . \quad . \quad . \quad . \quad (9)$$
$$b_2 + c_3 = 2\{-[\mathrm{B_2C_3}] + k\} \quad . \quad . \quad . \quad (10)$$

If we have started by assuming $a_1 \ a_2 \ a_3 \ a_4$ etc., then we deduce the b's; and equations (9) and (10) serve to deduce the c's. From this

point all the other corrections in two diagonals will be modified by the quantity k, with alternately positive and negative signs. Thus if the two diagonals originally read

$$\begin{array}{cccc} 0{\cdot}0 & & & 0{\cdot}0 \\ & +6{\cdot}0 & +4{\cdot}2 & \\ & -9{\cdot}4 & -0{\cdot}8 & \\ +13{\cdot}2 & & & -2{\cdot}8 \end{array}$$

we can add the same quantity, say $+5{\cdot}1$, to both $-9{\cdot}4$ and $-0{\cdot}8$ provided we subtract it from $+13{\cdot}2$ and $-2{\cdot}8$, and add it to the terms next to these, and so on. The few terms given above will then read

$$\begin{array}{cccc} 0{\cdot}0 & & & 0{\cdot}0 \\ & +6{\cdot}0 & +4{\cdot}2 & \\ & -4{\cdot}3 & +4{\cdot}3 & \\ +8{\cdot}1 & & & -7{\cdot}9 \end{array}$$

28. It is clear that we have ample facilities for devising sets of corrections to the original places which shall make overlapping plates accordant; the question now seems to be how to limit our choice. It seems probable that in practice the limitations will arise from the necessity of satisfying known conditions for particular stars. Running down a diagonal in the manner indicated, we shall come to stars with well-determined places and known proper motions, and the proper correction to the adopted place will be known within narrow limits. Such a method cannot be discussed in general terms, but will be special in each particular case; though experience of several cases may suggest some general remarks. But it seems clear that it will serve a useful purpose to calculate the "inconsistency" of every plate, to exhibit the results in diagrammatic form, and then formulate an empirical set of corrections. This work has been put in hand at Oxford, but will naturally take a little time to complete.

29. Meanwhile, it seems desirable to publish this note for the following reason. We have formed at Oxford ledgers of the corrections to the standard stars from all the plates on which they occur, and have done much work in examining the larger discrepancies. In many cases, of course, they have been traced to mere numerical errors (comparatively seldom to errors in measurement), but a sensible number have persistently defied explanation, and yet are so large as to suggest an error. It is now realised that these are due to "inconsistency" of the plate as above, and that the time spent in looking for errors or for better plate-constants was so much time lost. Before continuing the examination of large discrepancies, we propose now to tabulate the inconsistency for each plate and see how many discordances can be removed by suitable assumptions. It seems possible that a note of warning may save the time also of others who may have encountered similar puzzling discrepancies; and this is perhaps a fair excuse for publishing an incomplete investigation.

30. [Paragraphs 30 to end added December 3.] The above paragraphs (except 11 and 12 which have been expanded) were circulated in proof to several astronomers, and various criticisms received. From some of these I recognise that undue prominence has been given to the methods indicated in §§ 13 to 27. It was not intended to represent them as satisfactory (this should be clear from §§ 28 and 29); but in that case they might have been curtailed.

31. Further, it is well remarked by Professor Dyson that there may be independent methods of improving the plate-constants, *e.g.* we may be able to assume that a and e, the constants for scale value, are known from other plates; while b and d, the orientation constants, strengthen one another.

32. But the main point suggested for consideration is untouched. To make the point clear, I venture to restate it as follows, modifying the statement so as to take account of § 31.

(*a*) Find by any method the best linear solution for a plate. Denote the mean residuals in the four quarters by $x_1\ x_2\ x_3\ x_4$; and let

$$4\mathrm{I} = x_1 - x_2 + x_3 - x_4$$

Then I will be the same whatever linear solution is applied, or before any is applied at all; and if I is not zero, *no* linear solution will fit the *adopted* places.

(*b*) If we regard $x_1\ x_2\ x_3\ x_4$ as the proper corrections to the adopted places of the stars, then after applying them the new value of I will be zero; and we have a set of adopted places which can be fitted by a linear solution, viz. the solution already found.

(*c*) But we must take account of overlapping plates, and these do not indicate, in general, the same corrections: they give, say, x'_1, x'_2, x'_3, x'_4. Shall we then adopt as corrections

$$\tfrac{1}{2}(x_1 + x'_1),\ \tfrac{1}{2}(x_2 + x'_2),\ \tfrac{1}{2}(x_3 + x'_3),\ \tfrac{1}{2}(x_4 + x'_4)\,?$$

This procedure has the obvious advantage that we have only one correction for each star instead of two discordant ones; but it has the serious disadvantage that the inconsistency for each plate is not made zero.

Hence the corrections cannot be regarded as final. When we apply them to the adopted places and make new solutions for each plate we shall have the same situation as at first: the residuals from overlapping plates for the same star will not agree.

(*d*) We can, of course, repeat the process: again take means of the residuals, and again find new constants. We can make a third approximation and a fourth; and we have no guarantee that the process is not endless. A *necessary condition* of our having at last found satisfactory places is that the value of I for each plate should be zero, or at least so small that it can be explained in other ways—tilt, defective measures, etc. But, as indicated in § 12, we

have no guarantee that the inconsistencies will be reduced by this method: they may even be increased.

(*e*) We can only find by actual trial whether this is or is not the case. When we have tabulated the inconsistencies, we can very quickly test the effect of taking means of overlapping plates without performing any actual solutions. If the process is convergent, we can adopt the results to which it leads just as satisfactorily as we can take means between two plates in the first instance: if the process is not convergent, the first step is as wrong as any other; and we detect places where new meridian observations are *imperative* if any advance is to be made. Needless to remark, additional meridian observations will be always welcome and helpful; but they may not be immediately forthcoming.

Pogson's Observations of U Geminorum. Edited by H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In 1904 April Mr Joseph Baxendell, of Southport, put into the writer's hands a number of MSS., notebooks and charts, representing the work of his uncle, Mr N. R. Pogson, on Variable Stars. Mr Baxendell found that he had not the leisure necessary to arrange the material for publication, and expressed the hope that the editing of Knott's and Peek's observations (*Memoirs R.A.S.*, vols. lii. and lv.) might be extended to Pogson's. A small grant from the Government Grant Fund enabled me to get the original notebooks copied out, as a safeguard against loss or injury, and the charts were photographed by Mr Bellamy and the originals deposited with the Royal Astronomical Society. But as yet I have made but little headway with the actual copy for press. The material requires more study than might be expected, some of it being very scrappy.

2. A recent announcement of a prize question on the variable U Geminorum, by the University of Utrecht, produced some inquiries for Pogson's original observations of this curious variable, and turned my thoughts in a new direction. Instead of trying to deal with the whole material at once, which would require at least several weeks' continuous attention, could the observations of individual stars be published separately? As an experiment, the observations of U Geminorum have been collected as below, and their immediate publication will in any case serve the good purpose of putting this valuable early material in the hands of those now undertaking a discussion of this remarkable star.

3. Pogson began to look for this star soon after Hind's discovery on 1855 December 15, and his first observation of it was dated 1856 March 26, when he makes this note:—

> The variable subject to strange fluctuations at intervals of 6 to 15 seconds, and quite to the extent of 4 mags. The neighbouring small stars were steady, not at all twitching like